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**17年12月23日 Saturday**

**Sequences, A.S. and G.S. (1)  
數列、等差數列與等比數列(1)**

**Version: 2017.12.23.v338**

|  |  |  |
| --- | --- | --- |
| **Content** |  | |
| General Sequences | | 一般數列 | P. 4 | |
| Arithmetic Sequences | | 等差數列 | P. 10 | |
| Faster ways to find the common difference in A.S. | | 從等差數列中快速找公差 | P. 16 | |
| Questions related to the Number of Terms | | 與項數有關的題目 | P. 20 | |
| Properties of Arithmetic Sequences | | 等差數列的特性 | P. 24 | |
| Geometric Sequences | | 等比數列 | P. 29 | |
| Properties of Geometric Sequences | | 等比數列的特性 | P. 32 | |
| Faster ways to find the common ratio in G.S. | | 從等比數列中快速找公比 | P. 39 | |
| G.S. Questions related to Inequalities | | 與不等式有關的等比數列題目 | P. 43 | |
| Definition of Series | | 級數的定義 | P. 45 | |
| Sum of Arithmetic Sequences | | 等差數列之和 | P. 46 | |
| Gaussian Pairing on angles that form an Arithmetic Sequence | | 角度等差數列中的高斯配對 | P. 51 | |
| Sum of Geometric Sequences | | 等比數列之和 | P. 53 | |
| Sum to infinity of Geometric Sequences | | 等比數列無限項之和 | P. 59 | |
| Figures: Number of Dots | | 圖形題:圖案的點子 | P. 63 | |
| Appendix | | 附錄 | P. 70 | |
| SOLUTIONS | | 題解 | P.72 | |

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| --- | --- | --- | --- |
| **Number and Algebra Strand (SENIOR SECDONARY)**  **數與代數範疇(高中)** | | | |
| **Learning Unit**  **學習單位** | **Learning Objective**  **學習重點** | **Time**  **時間** | **Remarks**  **注釋** |
| 7. Arithmetic and geometric sequences and their sums  等差數列與等比數列及其求和法 | 1. understand the concept and the properties of arithmetic sequences   理解等差數列的概念及其性質   1. understand the general term of an arithmetic sequence   理解等差數列的通項   1. understand the concept and the properties of geometric sequences   理解等比數列的概念及其性質   1. understand the general term of a geometric sequence   理解等比數列的通項   1. understand the general formulae of the sum of a finite number of terms of an arithmetic sequence and a geometric sequence and use the formulae to solve related problems   理解等差數列和等比數列的有限項求和公式及使用該公式解有關問題   1. explore the general formulae of the sum to infinity for certain geometric sequences and use the formulae to solve related problems   探究某些等比數列的無限項求和公式及使用該公式解有關問題   1. solve related real-life problems   解有關現實生活中的應用題 | 17h | Some properties of arithmetic sequences :  等差數列的性質：   * Ifis an arithmetic sequence, then is an arithmetic sequence   若為等差數列,則亦為等差數列  Some properties of geometric sequences :  等比數列的性質：   * Ifis a geometric sequence, then is a geometric sequence   若為等比數列,則亦為等比數列  Example: geometry problems related to the sum of arithmetic or geometric sequences  例如︰涉及等差數列或等比數列求和的幾何題。  Example: geometry problems related to infinite sum of geometric sequences.  例如︰涉及等比數列的無限項求和的幾何題。  Examples:  Interest, growth or depreciation.  例如︰利息、增長或折舊。 |

**Main Concept 1:**

**General Sequences 一般數列**

|  |  |
| --- | --- |
| A **sequence** is a set of numbers (or patterns) arranged in a particular order.  Each number in the sequence is called a term of the sequence, the ***n*-th term of the sequence** is called the **general term**. | **數列**是一組按順序排列的數。  數列中的每個數被稱為項，該數列的**第*n*項**被稱為**通項**。 |

**Different kinds of Special Sequences 各種特殊數列:**

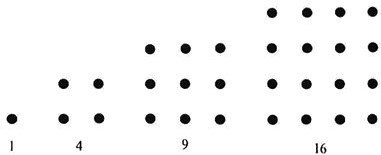
|  |  |
| --- | --- |
| 1. **Arithmetic Sequences** 2. **Geometric Sequences** 3. Square Sequences 4. Cubic Sequences 5. Triangular Sequences 6. Fibonacci Sequences 7. Other Special Sequences | 1. **等差數列** 2. **等比數列** 3. 正方形數列 4. 立方數列 5. 三角形數列 6. 斐波那契數列 7. 其他特殊數列 |

**Remarks 備註:**

|  |  |
| --- | --- |
| 1. General sequences appear in Paper II where you are provided with a sequence of numbers or patterns (usually dots or graphs). 2. Paper 2 skills:   To find the general term, you may **substitute the first few terms** and check if they can fit each of the numbers provided or derive it by **the method of induction**. (e.g. 1, 6, 11, 16, …）   1. Be careful: some general terms of special sequences involve (e.g. 5, 12, 21, 32, 45...) 2. Be careful: the **sign** of some general terms of the sequences. You may have to multiply (e.g. 3, 4, 5, 6, …) | 1. 一般數列出現在卷2，卷2問題提供有規律的數字或圖案（通常是點或圖表）。 2. 要找通項，您可以**代數字**來檢查，答案提供的**通項**是否滿足每一個提供的數字，或用**列舉法**(e.g. 1, 6, 11, 16, …）。 3. 小心: 部分數列的**通項**會出現   (e.g. 5, 12, 21, 32, 45,...)   1. 小心: 部分數列的**通項**的**正負**。你可能需要乘上   (e.g. 3, 4, 5, 6, …) |

|  |  |
| --- | --- |
| **Exam Analysis 考試分析:** | |
| 1. You should classify the types of sequences first:   A.S?  G.S?  Others?   1. With the types of sequences, we may make use of the concepts and skills.   ? A.S? G.S? Others? | 1. 你應該首先分辨數列的種類:   A.S?  G.S?  其他?   1. 根據該類型的數列，利用概念及技巧去作答。   ? A.S? G.S? 其他? |

**Square Sequences正方形數列**



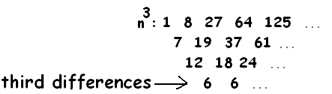
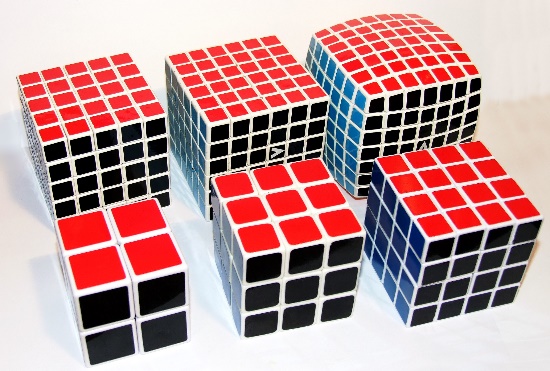
**Example 1:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Sequence數列 | | 1, 4, 9, 16, 25, … | | | | | | | | General Term *T*(*n*):  通項 |
|  | *n* | 1 | 2 | 3 | 4 | 5 |  | *m* |  |
| *T*(*n*) |  | 4 | 9 | 16 | 25 |  |  |
| *T*(*n*) |  |  |  |  |  |  |  |

**Example 2:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Sequence數列 | | 4, 9, 16, 25, 36, … | | | | | | | | General Term *T*(*n*):  通項 |
|  | *n* | 1 | 2 | 3 | 4 | 5 |  | *m* |  |
| *T*(*n*) | 4 | 9 | 16 | 25 | 36 |  |  |
| *T*(*n*) |  |  |  |  |  |  |  |

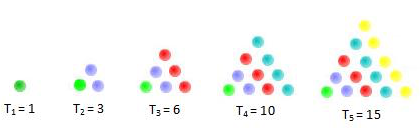
**Cubic Sequences 立方數列**



**Example 1:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Sequence數列 | | 1, 8, 27, 64, 125, … | | | | | | | | General Term *T*(*n*):  通項 |
|  | *n* | 1 | 2 | 3 | 4 | 5 |  | *m* |  |
| *T*(*n*) |  | 8 | 27 | 64 | 125 |  |  |
| *T*(*n*) |  |  |  |  |  |  |  |

**Triangular Sequences三角形數列**

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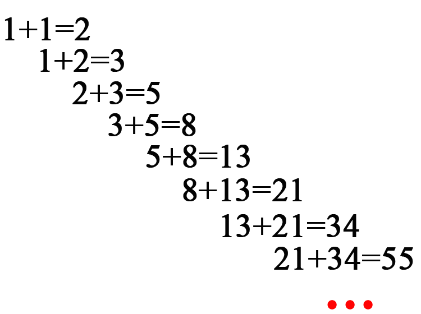
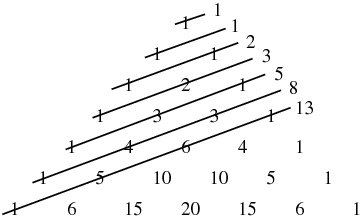
**Example1:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Sequence數列 | | 1, 3, 6, 10, 15, … | | | | | | | | General Term *T*(*n*):  通項  = |
|  | *n* | 1 | 2 | 3 | 4 | 5 |  | *n* |  |
| *T*(*n*) |  | 3 | 6 | 10 | 15 |  |  |
| *T*(*n*) | 1 |  |  |  |  |  |  |

**Fibonacci Sequences 斐波那契數列**

**Example 1:**

|  |
| --- |
| Sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, …  數列 |

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**Main Concept 1- Checkpoint 1:**

|  |  |
| --- | --- |
| Find the general term of the following sequences and write down the values of *T*(*n*):   1. 1, | 找出下列數列的通項, 並寫出*T*(*n*)的數值。 |

**Main Concept 1- Checkpoint 2:**

|  |  |  |  |
| --- | --- | --- | --- |
| Which of the following may represent the term of the sequence ? | | 下列何者可表示數列的第*n*項? | |
| A. | B. | C. | D. |

Checkpoint 1

(a) (b) or (c)

(d) (e) (f)

Checkpoint 2

C

**Main Concept 1- Checkpoint 3 (2014DSE Paper 2 Q14):**

|  |  |  |  |
| --- | --- | --- | --- |
| Let be the term of a sequence. If = 7, = 63 and = + for any positive integer *n*,then  *=* | | 設為某數列的第*n*項。若 = 7 及= 63 及對任意正整數*n*， = + ，則 | |
| A. 56 | B. 70 | C. 91 | D. 119 |

**Main Concept 1- Checkpoint 4 (2009CE Paper 2 Q12):**

|  |  |  |  |
| --- | --- | --- | --- |
| In the following sequence, the 1st term, the 2nd term and the 3rd term are 1, 2 and 3 respectively. For any positive integer *n*, the term is the sum of the term, the term and the term. Find the 9th term of the sequence. | | 在以下的數列中，第1項、第2項及第3項分別為1、2及3。對任意正整數*n*，第(項為第(項、第(項與第項之和。求該數列的第9 項。 | |
|  | | | |
| A.51 | B. 68 | C. 125 | D. 230 |

**Main Concept 1- Checkpoint 5 (SPDSE Paper 2 Q11):**

|  |  |  |  |
| --- | --- | --- | --- |
| Let be the term of a sequence. If , and for any positive integer *n*, then | | 設*an*為某數列的第*n*項。若， 及對任意正整數*n*，，則 | |
| A. 13 | B. 157 | C. 254 | D. 411 |

Checkpoint 3

D

Checkpoint 4

C

Checkpoint 5

C

**Main Concept 2:**

**Arithmetic Sequences 等差數列**

|  |  |
| --- | --- |
| Arithmetic Sequence is a sequence which has a **common difference (*d*)** between two consecutive terms.  (i.e. ) | 任何兩個連續之**差相等**稱為等差數列。  (i.e. ) |

**Example 1:**

|  |  |
| --- | --- |
| Sequence: 3, 5, 7, 9, 11, …  數列 | General Term *T*(*n*):  通項 |

**Example 2:**

|  |  |
| --- | --- |
| Sequence: , , , , , …  數列 | General Term *T*(*n*): *n* +  通項 |

How to find the general term of an A.S.? 如何找到A.S.的通項?

**Method 1(memorizing the formula):**

|  |  |
| --- | --- |
| where *a* = the first term and *d* = common difference  First term = *T* (1) = *a*  Common difference = *d =* | 其中 *a* = 首項，*d* = 公差  首項 = *T* (1) = *a*  公差 = *d =* |

**Method 2 (using features of A.S.):**

|  |  |
| --- | --- |
| Consider an A.S. | 考慮等差數列 |
| e.g. 1, , ,, …  + 1st order differences are equal | |
| 1. (+6) implies that *T*(*n*) includes 6*n* 2. Sub *n* = 1 or 2 or 3 … to find the constant:   Set . Then .    General Term | 1. (+6) 暗示*T*(*n*) 必定有6*n* 2. 代 *n* =1或2或3 ... 找出常數:   設 ，則。 |

**Graphically 圖像上**

|  |
| --- |
|  |

In other word,

|  |  |
| --- | --- |
| e.g.  If *T*(*n*) = 6*n* – 5,  **the coefficient of *n*** is the common difference,  i.e. *d* = 6  e.g.  If *T*(*n*) = + 1000,  common difference = *k* | e.g.  若*T*(*n*) = 6*n* – 5,  ***n*的係數**是公差,  即 *d* = 6  e.g.  若*T*(*n*) = + 1000，  公差 = *k* |

**Exam Tips:**

**A.S.多考慮加減**

**Main Concept 2- Checkpoint 1:**

|  |  |  |  |
| --- | --- | --- | --- |
| Given the first term and the common difference of an arithmetic sequence, write down its general term .  (Try Method 1 and Method 2) | | | 已知某等差數列的首項及公差，寫出它的通項  (試運用方法1及方法2) |
|  | First term 首項 | Common difference 公差 | General Term 通項 |
| e.g. |  |  | **Method 1:**  into formula:  **Method 2:**  must have . So .  . Then. |
| (a) |  | 7 |  |
| (b) |  |  |  |
| (c) | *k* | 2*k* |  |

Checkpoint 1

(a)

(b)

(c)

**Main Concept 2- Checkpoint 2:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Complete the table below. | | | 完成下表 | |
|  | General Term of Arithmetic sequence  等差數列的通項 | First Term  首項 | Common difference  公差 | The 99th term  第99項 |
| e.g. | 7*n* + 9 |  | 7 |  |
| (a) |  |  |  |  |
| (b) |  |  |  |  |
| (c) |  |  |  |  |

**Main Concept 2- Checkpoint 3:**

|  |  |  |  |
| --- | --- | --- | --- |
| The term of the arithmetic sequence 17, 14, 11, is | | 等差數列17, 14, 11, 的第*n*項為 | |
| A. 14 + 3*n* | B. 17 + 3*n* | C. 17 – 3*n* | D. 20 – 3*n* |

**Main Concept 2- Checkpoint 4:**

|  |  |  |  |
| --- | --- | --- | --- |
| If the common difference of the arithmetic sequence is *d*, then the common difference of the arithmetic sequence is | | 若等差數列 的公差為*d*，則等差數列  的公差為 | |
| A. -2*d* | B. *d* | C. -2 | D. 7 |

Checkpoint 2

(a)

(b)

(c)

Checkpoint 3

D

Checkpoint 4

A

**Main Concept 2- Checkpoint 5:**

|  |  |  |  |
| --- | --- | --- | --- |
| Consider the arithmetic sequence 1, 2, 5, , if the *k*th term of the sequence is 56, find *k*. | | 考慮一等差數列 1, 2, 5, , 若該數列的第*k*項為56, 求*k*。 | |
| A. 19 | B. 20 | C. 21 | D. 22 |

**\**

**Main Concept 2- Checkpoint 6 (1988CE Paper2Q38):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *p*, *q*, *r*, *s* are in A.P. If *p* + *q* = 8 and *r* + *s* = 20, then the common difference is | | | *p*, *q*, *r*, *s*成等差級數。若*p + q* = 8 及*r* + *s* = 20，  該公差為 | | |
| A. 3 | B. 4 | C. 6 | | D. 7 | E. 12 |

Checkpoint 5

B

Checkpoint 6

A

**Main Concept 3:**

**Faster ways to find common difference in A.S. 從等差數列中快速找公差**

|  |  |
| --- | --- |
| **Exam Analysis 考試分析:** | |
| Given any two terms of A.S, be they constant or unknown, find the **unknown** or **common difference** or **general term**. | 已知等差數列中的任何兩項，話之佢常數還是未知數，求當中**未知數**或**公差**或**通項**。 |
| Formula: or | |

Think How is common difference *d* related to the graph in **Main Concept 2**?

**Example 1 (Solving the unknown and general term解未知數及找通項):**

|  |  |
| --- | --- |
| Given an A.S. *a*, *b*, 6, *c*, *d*, 33, …  (a) Without finding the general term, find *a + b + c + d*.  (b) Write down the general term. | 已知一等差數列為*a*, *b*, 6, *c*, *d*, 33, …  (a) 在不找出通項的情況下，求*a + b + c + d* 。  (b) 寫下通項。 |

**Solution to Example 1:**

|  |  |
| --- | --- |
| (a) | (a) |
| (b) | (b) 通項: |

**Example 2 (adding/inserting numbers加入數字):**

|  |  |
| --- | --- |
| Given two numbers 8 and 40, insert 3 numbers between them such that these five numbers form an arithmetic sequence. | 於8和40之間加入三個數字，使得這五個數字組成一等差數列。 |

**Solution to Example 2:**

|  |  |
| --- | --- |
| Let *d* be the common difference of the sequence  The required three numbers are  , ,  i.e. 4, 16, 28 | 設 *d* 為該數列的公差  所求的三個數字為  , ,  i.e. 4, 16, 28 |

**Remark:**

|  |  |
| --- | --- |
| This method can replace forming simultaneous equations to solve for the common difference and the first term. | 此方法能代替使用聯立方程去求公差及首項。 |

**Main Concept 3 - Checkpoint 1 (DSESP Paper 2 Q36):**

|  |  |  |  |
| --- | --- | --- | --- |
| If the term and the term of an arithmetic sequence are 18 and respectively, then the 2nd term of the sequence is | | 若某等差數列的第 3 項及第 6 項分別為 18 及，則該數列的第 2 項為 | |
| A. -8 | B. 10 | C. 26 | D. 34 |

**Main Concept 3 - Checkpoint 2 (2000CE Paper 2 Q15):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| The and terms of an arithmetic sequence are 2 and 29 respectively. The term of the sequence is | | | 某等差數列的第 1 和第10 項分別是2 和 29 。該數列的第20項是 | | |
| A. 56 | B. 58 | C. 59 | | D. 60 | E. 62 |

**Main Concept 3- Checkpoint 3:**

|  |  |
| --- | --- |
| Given two numbers 99 and , insert 5 numbers between them such that these seven numbers form an arithmetic sequence. Find the five numbers inserted. | 於99和 之間加入五個數字，使得這七個數字組成一等差數列。求該五個數字。 |

Checkpoint 1

C

Checkpoint 2

C

Checkpoint 3

81, 63, 45, 27, 9

**Main Concept 3- Checkpoint 4:**

|  |  |  |  |
| --- | --- | --- | --- |
| If three numbers are inserted between 11 and 23 such that these five numbers form an arithmetic sequence, then the sum of the three numbers inserted is | | 若在11與23之間插入三個數使得該五個數組成一等差數列，則該三個數之和為 | |
| A. 33 | B. 51 | C. 56 | D. 59 |

**Main Concept 3 - Checkpoint 5 (1983CE Paper2Q9):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| The sixth term and the eleventh term of an arithmetic progression are 10 and 30 respectively. The first term is | | | 在某等差級數，第六項及第十一項分別為10及30。第一項為 | | |
| A. 14 | B. | C. 10 | | D. 50 | E. 54 |

**Main Concept 3- Checkpoint 6:**

|  |  |
| --- | --- |
| In an arithmetic sequence, the and terms of an arithmetic sequence are 24 and 42 respectively. | 在一等差數列中，第12項是24而第18項是42。 |
| 1. Find the general term *T*(n). 2. Determine whether 105 is a term of the arithmetic sequence. | 1. 求該數列的通項 *T*(n)。 2. 判斷 105是否該數列的其中一項。 |

Checkpoint 4

B

Checkpoint 5

B

Checkpoint 6

(a) 3*n* – 12

(b) Yes

**Main Concept 4:**

**Questions related to Number of Terms與項數有關的題目**

|  |  |
| --- | --- |
| **Exam Analysis 考試分析:** | |
| 1. You can use this formula: | 1. 你可使用這公式 |
|  | |
| 1. Some question types related to number of terms:   Type 1: Find the number of terms  Type 2: Find the number of multiples  Type 3: Find the greatest/least possible values of *k*  such that or  Type 4: Find the first positive or negative term | 2. 部分與項數有關的題種:  題種1: 求項數  題種2: 求倍數的數目  題種3: *k*的最大或最少可能值使得  或  題種4: 求第一個正數/負數項 |

**Type 1: Find the number of terms 求項數**

1. **Use or**
2. **Use**  [Teach Later!!!]

**Example 1:**

|  |  |
| --- | --- |
| Consider an arithmetic sequence 9, 18, 27, 36, , 81, find the number of terms. | 考慮一等差數列9, 18, 27, 36, , 81, 求項數。 |

**Solution to Example 1:**

|  |  |
| --- | --- |
| Let *d* be the common difference  *d* = 18 – 9 = 9  *n* = | 設*d*為公差  *d* = 18 – 9 = 9  *n* = |

**Type 2: Find the number of multiples 求倍數的數目**

Use

**Example 2 (For a more general case, we will discuss it in class):**

|  |  |
| --- | --- |
| It is known that 9 and 27 are multiples of 3. Find the number of multiples of 3 between 9 and 27 (including 9 and 27). | 已知 9和27是3的倍數。求 9及27之間 (包括9與27) 的3的倍數的數目。 |

**Solution to Example 2:**

|  |  |
| --- | --- |
| *n* = | *n* = |

**Type 3: Find the greatest/least possible values of *k*** such that or

Find the general term 找出通項

**]**

**Example 3:**

|  |  |
| --- | --- |
| Consider an arithmetic sequence 7, 0, 7, 14, …, find  the least possible value of *k* such that . | 考慮一等差數列 7, 0, 7, 14, …, 求*k*的最小可能值使得 。 |

**Solution to Example 3:**

|  |  |
| --- | --- |
| **Remarks:**   1. For the number of terms, the answer must be an integer. 2. Inequality: | **備注:**   1. 項數的答案必定為整數 2. 不等式:負數 不等式符號” >” 倒轉 |

**Type 4: Find the first positive or negative term 求第一個正數/負數項**

|  |  |
| --- | --- |
| Find the general term:  First positive number: set the general term  First negative number: set the general term | 找出通項:  第一個正數: 設通項  第一個負數: 設通項 |

**Example 4:**

|  |  |
| --- | --- |
| Consider an arithmetic sequence 77, 66, 55, 44, …, find the first positive term in the sequence. | 考慮一等差數列77, 66, 55, 44, …,  求首個正值項。 |

**Solution to Example 4:**

|  |  |
| --- | --- |
| The 1st positive term = | 首個正值項 = |

**Main Concept 4 - Checkpoint 1**

|  |  |
| --- | --- |
| Given the arithmetic sequence 148, 137, 126, …, find the number of positive terms and the first negative term. | 已知等差數列148, 137, 126, …, 求正數項的數目及首個負數項。 |

**Main Concept 4 - Checkpoint 2:**

|  |  |
| --- | --- |
| Find the number of terms between 100 and 200 of the arithmetic sequence 48, 57, 66, … | 求等差數列48, 57, 66, …在100至200之間的項數。 |

**Main Concept 4- Checkpoint 3:**

|  |  |
| --- | --- |
| Find the number of multiples of 7 between 9 and 143. | 求 9及143之間的7的倍數的數目。 |

**Main Concept 4- Checkpoint 4:**

|  |  |
| --- | --- |
| The general term of an arithmetic sequence is .   1. Find the common difference of the sequence. 2. Find the greatest possible value of *k* such that > 37. | 某等差數列的通項為。   1. 求該數列的公差。 2. 求*k*的最大可能值使得> 37。 |

**Main Concept 5:**

**Properties of Arithmetic Sequences 等差數列的特性**

|  |
| --- |
| 1. **Arithmetic Mean等差中項**   Given an arithmetic sequence(等差數列) , we have  ,  where *k* is a positive integer (正整數),  For *k* =1, we have   1. **Arithmetic Mean等差中項** 2. If (definition), then form an arithmetic sequence. 3. If for every positive integer *n* (definition), then form an arithmetic sequence.   **How to check whether a sequence is an A.S :**  **Check whether**   1. If form an arithmetic sequence(等差數列),   then also form an arithmetic sequence, where *k* and *c* are constants(常數).   1. The sum (和) of two arithmetic sequences is an arithmetic sequence (等差數列).   **Proof:** |

**Example 1:**

|  |  |
| --- | --- |
| Show that is an A.S. | 證明 為等差數列。 |

**Solution to Example 1:**

|  |  |
| --- | --- |
|  |  |

**Example 2:**

|  |  |
| --- | --- |
| Explain whether is an A.S. | 解釋 是否等差數列。 |

**Solution to Example 2:**

|  |  |
| --- | --- |
|  |  |

**Remark :** Or you may simply find an counterexample by showing .

**Example 3: (to be discussed in class)**

|  |  |
| --- | --- |
| If *a, b, c, d* are consecutive terms of an arithmetic sequence, which of the following **must** be true? | 若*a, b, c, d* 為一等差數列的連續項，則下列何者**必為**正確？ |
| 2. 2*d*, 2*c*,2*b*, 2*a* are consecutive terms of an arithmetic sequence 3. *c*, *d*, *a*, *b* are consecutive terms of an arithmetic sequence | I.    III. 2*d*, 2*c*,2*b*, 2*a*為一等差數列的連續項。  IV. *c*, *d*, *a*, *b*為一等差數列的連續項。 |

**Main Concept 5- Checkpoint 1:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 4 distinct numbers *a, b, c* and *d* are in arithmetic sequence. Which of the following is/are also in arithmetic sequence? | | | 4個相異數字*a, b, c* 及*d* 為一等差數列。下列何者必為等差數列？ | | |
|  | | |  | | |
| A. I only | B. I and II only | C. I and III only | | D. II and III only | E. All of the above |

**Main Concept 5- Checkpoint 2:**

|  |  |  |  |
| --- | --- | --- | --- |
| If 6, *x,* 24, … is an arithmetic sequence, then *x* = | | 若6, *x,*24,…為一等差數列，則*x* = | |
| A. 15 | B. 12 | C. 12 | D. 15 |

**Main Concept 5- Checkpoint 3:**

|  |  |  |  |
| --- | --- | --- | --- |
| If three numbers are inserted between 11 and 23 such that these five numbers form an arithmetic sequence, then the sum of the three numbers inserted is | | 若在11與23之間插入三個數使得該五個數成一等差數列，則該三個數之和為 | |
| A. 33 | B. 51 | C. 56 | D. 59 |

Checkpoint 1

B

Checkpoint 2

D

Checkpoint 3

B

**Main Concept 5- Checkpoint 4 (1985CE Paper2Q39):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Three distinct numbers *x*, *y* and *z* are in arithmetic progression. Which of the following is/are also in arithmetic progression?   1. *x* + 10, *y* + 10, *z* + 10 2. 10*x*, 10*y*, 10*z* 3. *x*2, *y*2, *z*2 | | | 三個不同的數字*x*、*y*及 *z*成等差級數。以下哪一個選項可以成等差級數？   1. *x* + 10, *y* + 10, *z* + 10 2. 10*x*, 10*y*, 10*z* 3. *x*2, *y*2, *z*2 | | |
| A. I and II only | B. I and III only | C. II and III only | | D. I, II and III | E. None of I, II and III |

**Main Concept 5- Checkpoint 5 (1998CE Paper2Q13):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| If *a*, *b*, *c*, *d* are consecutive terms of an arithmetic sequence, which of the following must be true?   1. *d*, *c*, *b*, *a* are consecutive terms of an arithmetic sequence | | | 若*a*, *b*, *c*, *d*是一等差數列的連續項，則下列何者必為正確？   1. *d*, *c*, *b*, *a* 是等差數列的連續項 | | |
| A. I only | B. I and II only | C. I and III only | | D. II and III only | E. I , II and III |

**Main Concept 5- Checkpoint 6 (2005CE Paper2Q42):**

|  |  |  |  |
| --- | --- | --- | --- |
| If four arithmetic means are inserted between 12 and 27, then the sum of the four arithmetic means is | | 若在12與27之間插入四個等差中項，則該四個等差中項之和為 | |
| A. 78 | B. 90 | C. 105 | D. 117 |

Checkpoint 4

A

Checkpoint 5

B

Checkpoint 6

A

**Main Concept 5- Checkpoint 7 (2008CE Paper2Q43):**

|  |  |  |  |
| --- | --- | --- | --- |
| Let *a*, *b* and *c* be positive integers. Which of the following must be arithmetic sequences? | | 設*a*、*b*及*c*均為正整數。下列何者必為等差數列？ | |
| A. I and II only | B. I and III only | C. II and III only | D. I , II and III |

**Main Concept 5- Checkpoint 8:**

|  |  |
| --- | --- |
| Explain whether a sequence of numbers , 4, , 6, … form an A.S. | 解釋數列, 4, , 6, …是否等差數列。 |

**Main Concept 5- Checkpoint 9:**

|  |  |
| --- | --- |
| If the sum of the third term and the ninth term of an arithmetic sequence is 108, find the sixth term of the sequence. | 若一等差數列的第三項與第九項之和為108，求該等差數列的的第六項。 |

Checkpoint 7

B

Checkpoint 8

No

Checkpoint 9

54

**Main Concept 6:**

**Geometric Sequences 等比數列**

|  |  |
| --- | --- |
| Geometric Sequence is a sequence which has a common ratio *r* between two consecutive terms. | 任何兩個連續項之比相等的數列稱為等比數列。 |

**General Term 通項 of a G.S.:**

First term = *a* =

*r* =

**G.S 的類別:**

(1) (e.g. 2, 4, 8, 16, 32, …)

(2) (e.g. 2, 4,8, 16, 32, …)

(3) (e.g. 2, 4, 8, 16, 32, …)

**Common Error 常犯錯誤:**

|  |  |
| --- | --- |
| Some G.S. might have 2 common ratios.  e.g.  . form a G.S.,  find the values of *a* and *b*. | 部分G.S. 會出現兩個公比。  e.g.  已知 為一等比數列，  求*a* 及*b*。 |
| In this case,  *a* can be  *b* can be | 這情況下，  *a* 能為或  *b* 能為 或 |
|  |
| Incorrect to leave only  as answers. | 答案只寫  並不正確。 |

|  |  |
| --- | --- |
| Add when dealing with the ***even-numbered*** roots | **雙數次方根**時記得開方要補。 |

Why?

**Main Concept 6- Checkpoint 1:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Write down the general term and of the following sequences. | | | 求下列等比數列的通項 及 。 | |
|  | Geometric sequence 等比數列 | General Term 通項 | |  |
| (a) | 3, 6, 12, 24, ... |  | |  |
| (b) | 5, 1, , , ... |  | |  |
| (c) |  |  | |  |

**Main Concept 6- Checkpoint 2:**

|  |  |
| --- | --- |
| How many terms are there in the geometric sequence  8, 4, 2, …, ? | 8, 4, 2, …, 這個等比數列共有多少項? |

Checkpoint 2

7

**Main Concept 7:**

**Properties of** **Geometric Sequences 等比數列的特性**

|  |
| --- |
| 1. **Geometric Mean等比中項**   Given form a geometric sequence (等比數列), we have  where *k* is a positive integer (正整數), then  For *k* =1,   1. **Geometric Mean等比中項** 2. If (definition) (定義),   then form a geometric sequence.   1. **How to check whether a sequence of numbers form a G.S. :**   Check whether   1. If form a geometric sequence, then    1. also form a geometric sequence    2. also form a geometric sequence   **BUT**   * 1. form an arithmetic sequence if each is positive;   2. DO **NOT necessarily** form a geometric sequence,   where in each of the above *k* is a non-zero constant.   1. The product (積) of two geometric sequences is a G.S.   **BUT**   1. **The sum (和) of two geometric sequences is not necessarily a G.S.**   Think When does the sum of 2 G.S. form a G.S. ? |

|  |  |
| --- | --- |
| **Exam Analysis 考試分析:** | |
| To determine whether a sequence is an A.S. or G.S. or neither of them, you can let the sequence and substitute the number into to check whether it is G.S for Paper 2. You should try at least two kinds of sequences to see whether both of them fulfil the definition of G.S. | 於卷2中，當判斷數列是不是 A.S., G.S. 或兩者皆不是，你可以設數列並代數字入, 檢查它是不是G.S.。你應嘗試兩種或以上的數列，並留意是不是兩者同時滿足G.S. 的定義。 |
| (1) (e.g. 2, 4, 8, 16, 32, …)  (2) (e.g. 2, 4, 8, 16, 32, …)  **(3) (e.g. 2, 4, 8, 16, 32, …),** | |
| Pay attention to (3).  留意(3)。 | |

**Main Concept 7- Checkpoint 1:**

|  |  |
| --- | --- |
| Find *x* in the following geometric sequences. | 求下列等比數列的*x*。 |
| (a) 12, *x*, 3  (b) *x*, 6,  (c) | |

**Main Concept 7- Checkpoint 2:**

|  |  |  |  |
| --- | --- | --- | --- |
| *k, 1, m, n* are 4 consecutive terms of a geometric sequence, which of the following must be true? | | *k, 1, m, n* 某等比數列的 4 個連續項，下列何者必然正確? | |
| I.  II.  III. | | I.  II.  III. | |
| A. I only | B. II only | C. I and III only | D. II and III only |

Checkpoint 1

(a) 6

(b)

(c) 4 or

Checkpoint 2

C

**Main Concept 7 - Checkpoint 3:**

|  |  |  |  |
| --- | --- | --- | --- |
| If *i, j, k* are in G.S., which of the following must be true? | | 若*i, j, k*為等比數列，下列何者必為正確? | |
| 1. , , are in G.S. 2. are in A.S. 3. are in G.S. | | 1. , , 為等比數列。 2. 為等差級數。 3. 為等比數列。 | |
| A. I only | B. III only | C.I and III only | D. II and III only |

**Q: Are in G.S.?**

**Main Concept 7 - Checkpoint 4 (1982CE Paper2Q37):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| If *x* ≠ 0, which of the following is/are geometric progression?   1. *x*, *x*2, *x*3, *x*4 2. *x*, 2*x*, 3*x*, 4*x*, 3. *x*, *−x*2, *x*3, *−x*4 | | | 若*x* ≠ 0，以下哪一個選項成等比級數？   1. *x*, *x*2, *x*3, *x*4 2. *x*, 2*x*, 3*x*, 4*x*, 3. *x*, *−x*2, *x*3, *−x*4 | | |
| A. I only | B. I and II only | C. I and III only | | D. II and III only | E. I, II and III only |

Checkpoint 3

B

Checkpoint 4

C

**Main Concept 7- Checkpoint 5 (1983CE Paper2Q9):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Three positive numbers *a*, *b* and *c* are in geometric progression. Which of the following are true?   1. , and are in geometric progression. 2. *a*2, *b*2, *c*2 are in geometric progression. 3. log *a*, log *b*, log *c* are in arithmetic progression. | | | 三個正數*a*, *b* 及*c*成等比級數。以下哪一個選項為正確？   1. , 成等比級數。 2. *a*2, *b*2, *c*2成等比級數。 3. log *a*, log *b*, log *c*成等差級數。 | | |
| A. I and II only | B. I and III only | C. II and III only | | D. I, II and III | E. None of them  三者皆非 |

**Main Concept 7- Checkpoint 6 (1984CE Paper2Q38):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Which of the following must be geometric progression(s)?   1. log 3, log 9, log 27, log 81 2. 0.9, 0.99, 0.999, 0.9999 3. 1, −3, 9, −27 | | | 以下哪一個選項必定為等比級數？   1. log 3, log 9, log 27, log 81 2. 0.9, 0.99, 0.999, 0.9999 3. 1, −3, 9, −27 | | |
| A. I only | B. III only | C. I and III only | | D. I and II only | E. I, II and III |

Checkpoint 5

D

Checkpoint 6

B

**Main Concept 7- Checkpoint 7 (1993CE Paper2Q37):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Given that the positive numbers *p*, *q*, *r*, *s* are in G.P., which of the following must be true?   1. are in G.P., where *k* is a non-zero constant. 2. *a p*, *a q*, *a r*, *as* are in G.P., where *a* is a positive constant. 3. log *p*, log *q*, log *r*, log *s* are in A.P. | | | 已知正數*p*, *q*, *r*, *s*成等比級數，下列何者必為正確？   1. 成等比級數，其中*k*為非零常數。 2. *a p*, *a q*, *a r*, *as*成等比級數，其中*a*為正常數。 3. log *p*, log *q*, log *r*, log *s*成等差級數。 | | |
| A. I only | B. II only | C. I and II only | | D. I and III only | E. I, II and III |

**Main Concept 7- Checkpoint 8 (1997CE Paper2Q37):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *a*, *b*, *c*, *d* are 4 consecutive terms of a geometric sequence. Which of the following must be true? | | | *a*, *b*, *c*, *d*為某等比數列的4個連續項。下列何者必為正確？ | | |
| A. II only | B. I and II only | C. I and III only | | D. II and III only | E. I, II and III |

Checkpoint 7

D

Checkpoint 8

E

**Main Concept 7- Checkpoint 9 (2006CE Paper2Q43):**

|  |  |  |  |
| --- | --- | --- | --- |
| Let *a, b* and *c* be positive integers. If , which of the following must be true?   1. log , log , log is an arithmetic sequence 2. *, ,*  is a geometric sequence 3. , , is a geometric sequence | | 設*a*、*b*及*c*均為正整數。若 ，下列何者必為正確？   1. log , log , log為一等差數列。 2. *, ,* 為一等比數列。 3. , , 為一等比數列。 | |
| A. I and II only | B. I and III only | C. II and III only | D. I , II and III |

**Main Concept 7- Checkpoint 10:**

|  |  |
| --- | --- |
| The general term of a sequence is given by  *=* . Show that the sequence is a geometric sequence. | 已知某數列的通項為 *=* 。證明該數列為等比數列。 |

**Main Concept 7- Checkpoint 11:**

|  |  |
| --- | --- |
| If the product of the third term and the ninth term of a geometric sequence is 49, find the sixth term of the sequence. | 若一等比數列的第三項與第九項之積為49，求該等比數列的的第六項。 |

Checkpoint 9

A

Checkpoint 11

**Main Concept 8:**

**Faster ways to find the common ratio in G.S. 從等比數列中快速找公比**

|  |  |
| --- | --- |
| **Exam Analysis 考試分析:** | |
| Given any two terms of G.S, find the **unknown** or **common ratio** or **general term**. | 已知等比數列中的任何兩項，求當中未知數或公比或通項。 |
| Formula: | |

**Example 1:**

|  |  |
| --- | --- |
| If 3, *a*, *b*, 81 is a G.S., then *a* + *b* = | 若3, *a*, *b*, 81為一等比數列，則*a* + *b* = |

**Solution to Example 1:**

|  |
| --- |
| 3  *a* = 3(3) = 9  *b* = 9(3) = 27 |

**Example 2:**

|  |  |
| --- | --- |
| Given G.S: , find . | 已知等比數列 , 求。 |

**Solution to Example 2:**

|  |
| --- |
|  |

**Example 3:**

|  |  |
| --- | --- |
| Given two numbers 1 and 2401, insert 3 numbers between them to form a G.S. | 已知兩個數字1及2401，在它們之間加入3個數字去組成一等比數列。 |

**Solution to Example 3:**

|  |  |
| --- | --- |
| Consider 1, \_\_, \_\_, \_\_, 2401.  By the formula: | 考慮 1, \_\_, \_\_, \_\_, 2401.  利用公式: |

Remarks:

|  |  |
| --- | --- |
| 1. You may as well form simultaneous equations to solve for the common ratio and the first term, but that is more tedious. 2. You may find the unknowns without finding the first term.   (see Example 2)   1. When the question is about G.S., consider the possibility of two outcomes.   (see Example 3) | 1. 使用聯立方程去求公比及首項較繁複。 2. 你可以在不知道首項的情況下找出未知數。   (留意例子2)   1. 處理G.S. 的題目時，考慮是否有兩種可能性。   (留意例子3) |

**Exam Tips:**

**G.S.多考慮乘除**

**Extension Problem 1:**

|  |  |  |  |
| --- | --- | --- | --- |
| *k, l, m, n* are 4 consecutive terms of a geometric sequence, which of the following must be true? | | *k, l*, *m, n* 某等比數列的 4 個連續項，下列何者必然正確? | |
| I. form a G.S.  II. form a G.S.  III. form a G.S. | | I. 為一等比數列  II. 為一等比數列  III. 為一等比數列 | |
| A. I and II only | B. I and III only | C. II and III only | D. I, II and III |

**Extension Problem 2:**

|  |  |  |  |
| --- | --- | --- | --- |
| *k, l, m, n* are 4 consecutive terms of an arithmetic sequence, which of the following must be true? | | *k, l*, *m, n* 某等比數列的 4 個連續項，下列何者必然正確? | |
| 1. form an A.S. 2. form an A.S. 3. form an A.S. | | 1. 為一等差數列 2. 為一等比數列 3. 為一等比數列 | |
| A. I and II only | B. I and III only | C. II and III only | D. I, II and III |

**Main Concept 8- Checkpoint 1:**

|  |  |
| --- | --- |
| Insert two numbers between 6 and , so that the four numbers form a geometric sequence. | 於6 及 中加入兩個數，使這四個數成為等比數列。 |

**Main Concept 8- Checkpoint 2:**

|  |  |
| --- | --- |
| Given a G.S. , find . | 已知一等比數列 及 ，求 。 |

**Main Concept 8- Checkpoint 3 (2011CE Paper2Q45):**

|  |  |  |  |
| --- | --- | --- | --- |
| The product of the 1st term and the 2nd term of a geometric sequence is 18 while the product of the 3rd term and the 4th term of the sequence is 288. The product of the 4th term and the 5th term of the sequence is | | 一等比數列的第1項與第2項之積為18，而該數列的第3項與第4項之積為288。該數列的第4項與第5項之積為 | |
| A. 576 | B. 864 | C. 1152 | D. 5184 |

Checkpoint 1

3,

Checkpoint 3

C

**Main Concept 9:**

**G.S. Questions related to Inequalities與不等式有關的等比數列題目**

|  |  |
| --- | --- |
| **Exam Analysis 考試分析:** | |
| 1. Notice mathematical symbols, e.g.   Greater than  Smaller than   1. Memorize the general term of G.S.: 2. When the unknown is the power, **take log**. 3. The number of terms must be a positive integer. 4. Inequality: reverse the inequality sign   log *k*, where 0 < *k* < 1, is negative. | 1. 留意數學符號, 例如:   大於   1. 牢記G.S. 的通項: 2. 未知數於次方時，用**log**。 3. 項數必須為正整數。 4. 不等式:負數不等式符號倒轉   log *k*, 其中0 < *k* < 1, 為負。 |

Example 1:

|  |  |
| --- | --- |
| Find the greatest value of *k* such that the term of the G.S. 2, 3, is smaller than 1000. | 找出*k*的最大值使得等比數列2, 3, 的第*k* 項少於1000。 |

Solution to Example 1:

|  |  |
| --- | --- |
| With the general term  The greatest value of *k* = 16 | 利用通項  *k*的最大值 = 16 |

**Main Concept 9 - Checkpoint 1:**

|  |  |
| --- | --- |
| A geometric sequence 8, 12, 18, 27, ... is given.   1. Find the term of the sequence. 2. Find the term which is just greater than 1500. Express your answer in scientific notation. | 已知某等比數列 8, 12, 18, 27, ...   1. 求數列的第8項。 2. 求僅僅大於1500 的項，以科學記數法表示答案。 |

**Main Concept 9 - Checkpoint 2:**

|  |  |
| --- | --- |
| The weight of Davis is 100 kg when he aged 21. It is known that each year his weight will be increased by 20%.   1. Find his weight when his age is 40. 2. Davis can make the ground break when his weight exceeds 5000 kg. At what age will he make the ground break? | 當穎豪為21歲時，他的重量為100kg。已知他的重量每年上升20%。   1. 求穎豪為40歲時的重量。 2. 穎豪能造成破土當他的重量超過5000 kg，他在哪歲的時候會造成破土? |

Checkpoint 2

(a) 3190kg

(b) 43

**Main Concept 10:**

**Definition of Series 級數的定義**

|  |
| --- |
| A series is the sum of a sequence,  一級數是數列之項的總和，以表示。    (首項)  Consider a sequence  考慮一數列  **Method 1:**  (\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_) – (\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  **Method 2:**  Let first term (首項) =  For A.S:  For G.S.: common ratio =  Number of terms  Use the corresponding (相應的)formulas to solve problems:  **A.S:**  **G.S:** |

**Main Concept 11:**

**Sum of Arithmetic Sequences 等差數列之和**

|  |  |
| --- | --- |
| Let *a*, *l*, *n* and *d* be the first term, the last term, the number of terms and the common difference respectively.  The sum of the first *n* terms of an A.S.,    can be found by:   1. Given the last term (*l*) 2. Without the last term (*l*)   where | 設*a*為首項，*l* 為尾項，*n* 為項數，*d* 為公差。  等差數列的總和，  能由以下方法找出:   1. 已知尾項(*l*) 2. 未知尾項(*l*)   其中 |

When to use this formula?

|  |  |
| --- | --- |
| Find the **total** / **sum** of … | 找出**總和** Sum of A.S. or G.S. |

**Main Concept 11- Checkpoint 1:**

|  |  |
| --- | --- |
| Find the sum of all the terms of the arithmetic sequences: 18, 29, 40, … to 77 terms | 求等差數列: 18, 29, 40, …共77項的總和。 |

**Main Concept 11- Checkpoint 2:**

|  |  |
| --- | --- |
| Find the sum of all the terms of the arithmetic sequences: , 4, , 29. | 求等差數列: , 4, , 29的總和。 |

Checkpoint 1

Checkpoint 2

(b)

**Main Concept 11- Checkpoint 3:**

|  |  |
| --- | --- |
| Given the general term of the arithmetic sequence , find the sum of the first 6 terms of the sequence. | 已知等差數列通項為 ，求該數列的首6項之和。 |

**Main Concept 11- Checkpoint 4 (1981CE Paper2Q34):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| The sum of the first five terms of an arithmetic progression is 15. If the fourth term is 7, the first term is | | | 該等差數列的首5項之和是15，如果該數列的第4項是7，該數列的第1項是 | | |
| A. 5 | B. 3 | C. −1 | | D. 1 | E. 10 |

**Main Concept 11- Checkpoint 5:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| If the sum to *n* terms of an A.P. is , find the term of the A.P. | | | 若某等差級數的*n*項和為 ，求第7項。 | | |
| A. 16 | B. 18 | C. 54 | | D. 70 | E. It cannot be found.  不可能求得 |

Checkpoint 3

Checkpoint 4

A

Checkpoint 5

A

**Main Concept 11- Checkpoint 6 (2007CE Paper2Q44):**

|  |  |  |  |
| --- | --- | --- | --- |
| Let be the term of an arithmetic sequence. If and , then | | 設為一等差數列的第*n*項。若 及 ，則 | |
| A. -52 | B. -26 | C. -23 | D. 139 |

**Main Concept 11- Checkpoint 7 (DSE PP Paper2Q35):**

|  |  |  |  |
| --- | --- | --- | --- |
| The term of a sequence is . If the sum of the first *m* terms of the sequence is less than 3000, then the greatest value of *m* is | | 某數列的第 項為 。若該數列的首 項之和小於3000，則*m*的最大值為 | |
| A. 52 | B. 53 | C. 56 | D. 57 |

Checkpoint 6

C

Checkpoint 7

A

**Main Concept 11- Checkpoint 8:**

|  |  |
| --- | --- |
| 1. Consider an arithmetic sequence 4, 7, 10, . If the sum of the first *n* terms of the sequence is 3150, find the value(s) of *n.* 2. The first term and the term of an arithmetic sequence are 24 and 32 respectively. If the sum of the first *n* terms is 75, find the value of *n.* | 1. 考慮等差數列4, 7, 10, 。若數列首*n*項的和是3150，求*n*的值。 2. 等差數列的首項及第9項分別為 24 及32。若首*n*項的和為75，求*n*的值。 |

**Main Concept 11- Checkpoint 9:**

|  |  |  |  |
| --- | --- | --- | --- |
| The first term and the common difference of an arithmetic sequence are 8 and 5 respectively. If , find . | | 某等差數列的首項及公差分別為8及5。若，求。 | |
| A. 17 | B. 18 | C. 24 | D. 25 |

Checkpoint 8

(a) 45

(b) 10

Checkpoint 9

B

**Main Concept 12:**

**Gaussian Pairing on angles that form an arithmetic sequence 角度等差數列中的高斯配對**

|  |  |
| --- | --- |
| **Exam Analysis 考試分析:** | |
| 1. Reverse the order of the sequence 2. Add the original sequence and the reversed   sequence   1. Use to find the number of terms 2. Ans = | 1. 把數列倒轉寫一次 2. 把原來的數列和倒轉的數列相加 3. 利用 找出項數 4. 答案 = |

**Example \*:**

|  |  |
| --- | --- |
| 1. Find the value of 2. Hence find the value of | 1. 求的值。 2. 由此，求的值。 |

**Solution to Example \*:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1. . 2. Let .   Let   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  |  |  |  | | + |  |  |  |  |  |  |  |  |  | |  |  |  | 1 |  |  |  |  |  |  | |  |  |  |  | | | | | | | |

**Main Concept 12 - Checkpoint 1:**

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
| A. 22 | B. 22.5 | C. 44.5 | D. 45 |

**Main Concept 12 - Checkpoint 2:**

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
| A. 10.5 | B. 11.5 | C. 12.5 | D. 13.5 |

Checkpoint 1

B

Checkpoint 2

B

**Main Concept 13:**

**Sum of Geometric sequence 等比數列之和**

|  |  |
| --- | --- |
| Let *a*, *n* and *r* be the first term, the number of terms and the common ratio respectively.  The sum to the first *n*th terms of G.S.,    can be found by:  S(*n*) = or S(*n*) = | 設*a*為首項，*n* 為項數，*r* 為公比。  等比數列的總和，    能由以下方法找出:  S(*n*) = or S(*n*) = |

When to use this formula?

|  |  |
| --- | --- |
| Find the **total** / **sum** of … Sum of A.S or G.S. | 找出**總和** ... A.S. 或 G.S. 之和 |

|  |  |
| --- | --- |
| **Exam Analysis 考試分析:** | |
| 1. Be careful of the **number of terms**   ***n* is not necessarily the number of terms.**  It may be (*n*1), (*n* + 2), (2*n*), etc.  **Example:**   |  | | --- | |  |  1. To find the number of terms, you can use   the formula  since the indices form an arithmetic sequence.  **Example:**   |  | | --- | | Consider 1, 3, 5, …, *n*  Corresponding no. of terms =  Total number of terms here is but it is unimportant here. |  1. You need not memorize both formulas.   You only need to memorize  S(*n*) = or S(*n*) = | 1. 小心**項數**，   ***n* 不一定是項數**  它可能是 (*n*1), (*n* + 2), (2*n*) 等等。  **例子:**   |  | | --- | |  |  1. 求項數，你可以使用公式：   因為指數是等差數列。  **例子：**   |  | | --- | | 考慮1, 3, 5, …, *n*  對應的項數 =  總項數是 ，但在這裡並不重要。 |  1. 你無須記兩條公式，   你只需要牢記  S(*n*) = or S(*n*) = |

**Specific Skills:**

|  |
| --- |
| You may classify 2 kinds of sum of G.S. after taking common factor:   1. (Hard) |

**Example 1:**

|  |
| --- |
|  |

**Example 2:**

|  |
| --- |
|  |

Given that G.S.

where *a* and *b* are distinct non-zero real numbers, find

a) **the common ratio** and

b) **the sum to *n* terms of the G.S.**

**Main Concept 13- Checkpoint 1 (2009CE Paper2Q43):**

|  |  |  |  |
| --- | --- | --- | --- |
| Let be the term of a geometric sequence. If and , which of the following must be true? | | 設　　為某等比數列的第*n*項。若 及，則下列何者必為正確？ | |
| A. I and II only | B. I and III only | C. II and III only | D. I, II and III |

**Main Concept 13- Checkpoint 2:**

|  |  |
| --- | --- |
| The general term of a geometric sequence is  .   1. Find the common ratio of the sequence. 2. Find the sum of the first 8 terms of the sequence. | 某等比數列的通項。   1. 求該數列的公比。 2. 求該數列的首8項之和。 |

**Main Concept 13- Checkpoint 3:**

|  |  |
| --- | --- |
| Let be the *n*thterm of a geometric sequence, where . It is given that and .   1. Find *.* 2. Find . | 設 為某等比數列的第*n*項，其中 。已知 及 。   1. 求 。 2. 求。 |

Checkpoint 1

A

**Main Concept 13- Checkpoint 4:**

|  |  |
| --- | --- |
| *, ,* , ... form a geometric sequence.   1. Find *.* 2. Let be the sum of the first *n* terms of the sequence. 3. Find . 4. If , find the least possible value of *k.* | *, ,* , ...為一等比數列。   1. 求 。 2. 設 為該數列的首*n*項之和。 3. 求 。 4. 若 ，求*k*的最小可能值。 |

**Main Concept 13- Checkpoint 5:**

|  |  |
| --- | --- |
| The sum of the first *n* terms of a geometric sequence is *.* It is given that and .   1. Prove that . 2. Find the common ratio of the sequence. 3. If , find the general term of the sequence. | 某等比數列的首*n*項之和為。已知 及 。   1. 證明。 2. 求該數列的公比。 3. 若 ，求該數列的通項 。 |

**Main Concept 13- Checkpoint 6:**

|  |  |
| --- | --- |
| *, ,* , ... form a geometric sequence.   1. Find *.* 2. Let be the sum of the first *n* terms of the sequence. 3. Find *.* 4. If , find the least possible value of *k.* | *, ,* , ... 為一等比數列。   1. 求。 2. 設 為該數列的首*n*項之和。 3. 求 。 4. 若 ，求*k*的最小可能值。 |

**Main Concept 13- Checkpoint 7:**

|  |  |
| --- | --- |
| Consider the geometric sequence 4, 12, 36,   1. Find the general term of the sequence. 2. If the sum of the first *k* terms of the sequence is greater than 30000, find the least possible value of *k.* | 考慮等比數列4, 12, 36,   1. 求該數列的通項 。 2. 若數列的首*k*項之和大於30000，求*k*的最小可能值。 |

**Main Concept 14:**

**Sum to infinity of Geometric Sequences 等比數列無限項之和**

|  |  |
| --- | --- |
| If , then sum to infinity  Otherwise, is undefined. | 若，則無限項之和。  否則， 未下定義。 |
| There is no sum to infinity for arithmetic sequence. | 等差數列沒有無限項之和。 |

Remark: do remember to check the condition for *r*.

**Example 1:**

|  |  |
| --- | --- |
| Find the sum to infinity of the G.S.: | 求等比數列無限項之和: |

**Solution to Example 1:**

|  |  |
| --- | --- |
|  |  |

Remarks:

|  |  |
| --- | --- |
| 1. Notice the differences between all the **positive** terms, all the **negative** terms and all the terms 2. If the question is about the sum to infinity | 1. 留意所有正數項，所有負數項，所有項的分別。 2. 題目關於無限項之和 |

**Main Concept 14- Checkpoint 1 (1981CE Paper2Q35):**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Which of the following can be summed to infinity?   1. The arithmetic progression 4, 3, 2, 1, …… 2. The geometric progression 27, 9, 3, 1, …… 3. The geometric progression 16, −8, 4, −2, …… | | | 以下哪一個選項可以相加至無限？   1. 等差級數4, 3, 2, 1, …… 2. 等比級數27, 9, 3, 1, …… 3. 等比級數16, −8, 4, −2, …… | | |
| A. II only | B. I and II only | C. I and III only | | D. II and III only | E. I, II and III only |

**Main Concept 14- Checkpoint 2:**

|  |  |
| --- | --- |
| Consider the geometric sequence *y, x,* 9, 3, …   1. Find *x* and *y.* 2. Find the sum to infinity of the geometric sequence. | 考慮等比數列*y, x,* 9, 3, ...   1. 求*x*及*y*。 2. 求該數列的無限項之和。 |

Checkpoint 1

D

**Main Concept 14- Checkpoint 3:**

|  |  |  |  |
| --- | --- | --- | --- |
| In a G.S., the sum to infinity is 10 and the second term is . Find the first term. | | 在一等比數列，無限項之和為10，第2項為 。  找出首項。 | |
| A. 14 only | B. 4 | C. 4 or 14 | D. 4 or 14 |

**Main Concept 14- Checkpoint 4 (2015DSE Paper2Q37):**

|  |  |  |  |
| --- | --- | --- | --- |
| Let be the nth term of a geometric sequence. If and , which of the following must be true?  I.  II.  III. | | 設 為一等比數列的第*n*項。若 及 ，則下列何者必為正確？  I.  II.  III. | |
| A. I only | B. II only | C. I and III only | D. II and III only |

Checkpoint 3

A

Checkpoint 4

D

**Main Concept 14- Checkpoint 5:**

|  |  |
| --- | --- |
| A geometric sequence is given: 20, *a*, *b*,,   1. Find the values of *a* and *b*. 2. Find the first negative term in the sequence which is greater than 0.00001. 3. Find the sum of all the negative terms which are greater than 0.00001. | 已知等比數列 20, *a*, *b*,,   1. 求*a* 及*b*的值。 2. 求數列中首個大於 0.00001 的負數項。 3. 求數列中所有大於 0.00001 的負數項的和。 |

**Main Concept 15:**

**Figures: Number of Dots 圖形題:圖案的點子**

|  |  |
| --- | --- |
| **Exam Analysis 考試分析:** | |
| 1. Determine the types of sequences:   A.S?  G.S?  Other special sequences?  (e.g. Fibonacci sequences)  Sum of A.S.?  Sum of G.S.?  Special sequences?   1. If A.S/G.S. handle the questions by concept. 2. Other special sequences:   Sum of A.S/G.S./Special sequences  Find the general term   1. The picture may not be provided. 2. **Paper 2 skills:**   To find the general term, you may substitute the first few terms and check if they can fit each of the numbers provided or derive it by the method of induction. (e.g. 1, 6, 11, 16, …)   1. Be careful: some general terms of special sequences involve (e.g. 5, 12, 21, 32, 45, ...) 2. Read the question carefully. It will provide   **hints** for you.   1. The last method: **count** it one by one | 1. 判斷數列的種類:   等差數列?  等比數列?  其他數列?  (e.g. 斐波那契數列)  等差數列之和?  等比數列之和?  特別數列?   1. 若為等差/等比數列 利用所學概念 2. 其他數列:   等差/等比數列之和/  找 通項   1. 圖案未必一定提供 2. **Paper 2 skills:**   要找通項，您可以**代數字**來檢查，答案提供的 通項是否滿足每一個提供的數字，或用**列舉法**(e.g. 1, 6, 11, 16, …)。   1. 小心: 部分數列的 通項 會出現   (e.g. 5, 12, 21, 32, 45, ...) 。   1. 小心留意題目，它會提供**提示**。 2. 最後方法: **一個個數** |

|  |  |
| --- | --- |
| **How to find the general term (of some special sequences)?**  **如何找出 (特別數列)** **的通項?** | |
| **Method 1 (Not recommended): By observing the pattern:** | 方法1(不推薦): 觀察規律 |
| e.g. 3, 8, 15, 24, ... | |
| **Method 2: By calculation** | **方法2: 計算** |
| e.g. 2, 7, 14, 23, ...  Consider the differences of the consecutive terms of the above sequence are  +5, +7, +9,  which form an A.S.  Sum of A.S. | e.g. 2, 7, 14, 23, ...  考慮上述數列的連續項的項差是  +5, +7, +9,  即組成等差數列。  等差數列的總和 |
| **Be careful: 2 is not the part of A.S.**  Number of terms is but not | **小心: 2 並不是A.S.的一部分**  項數是 |
| **Method 3: the differences form an A.S.** | **方法3: 利用項差組成A.S. 的技巧** |
| e.g. 2, 7, 14, 23, ...  Use ; the differences form an A.S. 項差組成A.S.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | 1 | 4 | 9 | 16 |  | | + | 1 | 3 | 5 | 7 |  | |  | 2 | 7 | 14 | 23 |  | | |

**Example 1(2006CE Paper2Q12)**

|  |  |  |  |
| --- | --- | --- | --- |
| In the figure, the 1st pattern consists of 3 dots. For any positive integer *n*, the pattern is formed by adding (2*n*) dots to the *n*th pattern. Find the number of dots in the 6th pattern. | | 圖中，第１個圖案包含3粒點子，對任意正整數*n*，第( *n* 1個圖案是由第*n*個圖案加上 (2*n*) 粒點子所組成。求第6個圖案的點子數目。 | |
|  | | | |
| A. 35 | B. 37 | C. 48 | D. 50 |

**Solution to Example 1:**

|  |
| --- |
|  |
| **Method 1 (By observing the algebraic pattern):** |
| **Method 2 (By calculation):** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Method 3 (By skills for the differences form an A.S.):**  The differences form an A.S. 差距組成A.S.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | 1 | 4 | 9 | 16 |  | | + | 2 | 4 | 6 | 8 |  | |  | 3 | 8 | 15 | 24 |  | |
| **Method 4 (By the method of induction):**    3, 8, 15, 24, 35, 48 |
| **Method 5 (By observing the geometric pattern):** |

**Main Concept 15- Checkpoint 1 (2007CE Paper2Q9):**

|  |  |  |  |
| --- | --- | --- | --- |
| In the figure, the 1st pattern consists of 4 dots. For any positive integer *n*, the pattern is formed by adding 4 dots to the *n*th pattern. Find the number of dots in the 9th pattern. | | 圖中，第1個圖案包含4粒點子。對任意正整數*n*，第(*n*+1) 個圖案是由第*n*個圖案加上4粒點子所組成。求第9個圖案的點子數目。 | |
|  | | | |
| 1. 36 | B. 40 | C. 81 | D. 100 |

**Main Concept 15- Checkpoint 2 (2008CE Paper2Q11):**

|  |  |  |  |
| --- | --- | --- | --- |
| In the figure, the 1st pattern consists of 10 dots. For any positive integer n, the pattern is formed by adding ( dots to the nth pattern. Find the number of dots in the 7th pattern. | | 圖中，第1個圖案包含10粒點子。對任意正整數*n*，第(個圖案是由第個圖案加上(粒點子所組成。求第7個圖案的點子數目。 | |
|  | | | |
| A. 50 | B. 65 | C. 82 | D. 101 |

Checkpoint 1

A

Checkpoint 2

C

**Main Concept 15- Checkpoint 3 (2011CE Paper2Q9):**

|  |  |  |  |
| --- | --- | --- | --- |
| In the figure, the 1st pattern consists of 4 dots. For any positive integer *n*, the pattern is formed by adding 3 dots to the *n*th pattern. Find the number of dots in the 8th pattern. | | 圖中，第1個圖案包含4粒點子。對任意正整數*n*，第(*n* + 1) 個圖案是由第*n*個圖案加上3粒點子所組成。求第8個圖案的點子數目。 | |
|  | | | |
| A. 22 | B. 25 | C. 28 | D. 31 |

**Main Concept 15- Checkpoint 4 (2012DSE Paper2Q12):**

|  |  |  |  |
| --- | --- | --- | --- |
| In the figure, the 1st pattern consists of 1 dot. For any positive integer n, the pattern is formed by adding *n* dots to the *n*th pattern. Find the number of dots in the 8th pattern. | | 圖中，第1個圖案包含1粒點子。對任意正整數*n*，第( *n* + 1 )個圖案是由第*n*個圖案加上*n*粒點子所組成。求第8個圖案的點子數目。 | |
|  | | | |
| A. 22 | B. 29 | C. 36 | D. 37 |

Checkpoint 3

B

Checkpoint 4

B

**Main Concept 15- Checkpoint 5:**

|  |  |  |  |
| --- | --- | --- | --- |
| The 1st pattern consists of 3 dots. For any positive integer *n*, the pattern is formed by adding 2*n* dots to the *n*th pattern. Find the number of dots in the 60th pattern. | | 第1個圖案包含3粒點子。對任意正整數*n*，第() 個圖案是由第*n*個圖案加上2*n*粒點子所組成。求第60個圖案的點子數目。 | |
| A. 3425 | B. 3543 | C. 3660 | D. 3663 |

Checkpoint 5

B

**Appendix**

**Solving Quadratic Inequalities Graphically**

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| **STEPS:**   1. Use the calculator to find the *x*-intercepts of . 2. Sketch . 3. Consider the inequality sign: or (above the *x*-axis)   or (below the *x*-axis)   1. Find the corresponding range of values of *x*. | | |
| **Type 1:**  SOL: or  SOL: or  SOL:  SOL:  If the *x*-intercepts are not rational numbers, use the quadratic formula: | **Type 2:**    SOL: all real numbers except  SOL: all real numbers  No solution  SOL: *x* = | **Type 3:**   |  | | --- | | The *x*-coordinate the vertex  The *y*-coordinate the vertex |   SOL: all real numbers  SOL: all real numbers  No solution.  No Solution. |
| Given ,  SOL: *x* =  No Solution.  SOL: All real numbers  SOL:all real numbers except 1 |
| How about ? Set your own questions! |

**Exponential Inequalities**

|  |  |
| --- | --- |
| All numerical answers are corrected to 3 s.f.   1. Laws of indices: 2. Properties of logarithms: 3. if *a* > 1, log *a* > 0;   if 0 < *a* < 1, log *a* < 0;  if *a* > 0, for all values of *a.*   1. Quadratic Formula:   If , then .   1. Quadratic inequalities:   Graphical representation [*x*-intercepts: ]  , then  , then  , then  , then | 答案以三位有效數字表示。   1. 指數法 : 2. 對數特性 : 3. 若 *a* > 1, log *a* > 0;   若 0 < *a* < 1, log *a* < 0;  若 *a* > 0, 對於所有*a.*   1. 二次公式:   若 , 則 .   1. 二次不等式:   圖像表示 [*x*-截距 : ]  , 則  , 則  , 則  , 則 |

|  |  |
| --- | --- |
| 1. Examples 例子   ­­­­­­­­­­­­­­­­­­­­­­­­­­­3*x* > 8  (3/4)*x* < 8  2(0.64*x*) + 7(0.8*x*) – 15 > 0    **Reminder:** never write 2(0.64*x*) as 1.28*x.* | **提示：**不要把2(0.64*x*)寫作1.28*x*。 |

**SOLUTION**

Main Concept 4:

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| **Checkpoint 1:** |
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| **Checkpoint 2:** |
|  |
| **Checkpoint 3:** |
| 19 |
| **Checkpoint 4:** |
| 1. Common Difference = = [114(*n*+1)] – (114*n*) = 4. 2. Note that   Hence the greatest possible value of *k* is 11. |

Main Concept 7:

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| **Checkpoint 10:** |
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Main Concept 8:

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| **Checkpoint 2:** |
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Main Concept 9:

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| **Checkpoint 1:** |
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Main Concept 11:

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| **Checkpoint 8:** |
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Main Concept 11:

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| **Checkpoint 9:** |
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Main Concept 13:

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| **Checkpoint 7:** |
|  |

Main Concept 14:

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| **Checkpoint 2:** |
|  |
| **Checkpoint 5:** |
|  |

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**Davis Tang**